

Spatially homogeneous anisotropic Bianchi Type I inflationary universe for barotropic fluid distribution with variable bulk viscosity

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Abstract

Spatially homogeneous anisotropic Bianchi Type I inflationary scenario for barotropic fluid distribution with variable bulk viscosity is investigated. To get the deterministic and realistic model, we have assumed $\rho \propto 3H^2$, $\zeta \propto \rho^{1/2}$ as considered by Barrow (1988) and Gron (1990) where ρ is the matter density, ζ the bulk viscosity, H the Hubble parameter and conservation equation $T_{i;j}^j = 0$ is taken into account. We find that bulk viscosity prevents the matter density to vanish. The model in general represents anisotropic universe but at late time, it isotropizes. The spatial volume increases with time representing inflationary scenario. The energy conditions as given by Kolassis et al. (1998), Chatterjee and Banerjee (2004) are discussed. The energy condition $\rho + p \geq 0$ is violated due to the presence of scalar field (ϕ) in inflationary universe.

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1. Introduction

The anisotropic models provide a systematic way to obtain cosmological models more general than Friedmann-Robertson-Walker (FRW) model. But FRW model are unstable near the singularity (Patridge and Wilkinson (1967)) and fail to describe early universe. Therefore, spatially homogeneous and anisotropic Bianchi Type I metric is undertaken to study the universe in its early stages of evolution of universe. The existence of anisotropic universe that approaches the isotropic phase is pointed out by Land and Magueijo (2005).

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The matter distribution is satisfactorily described by perfect fluid due to large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than perfect fluid. This is supported by the fact that when neutrino decoupling occurred, the matter behaved like viscous fluid in early stages of evolution of universe. Misner (1967, 1968) studied the effect of viscosity on the evolution of universe and suggested that strong dissipation due to the neutrino viscosity may considerably reduce the anisotropy. Anisotropic Bianchi Type I viscous cosmological models have been investigated by Belinski and Khalatnikov (1976). They pointed out that Bianchi Type I universe with viscous fluid will approach asymptotically isotropic steady state model. Heller (1978) analyzed viscous isotropic and anisotropic homogeneous cosmological models in relation to the weak, dominant and strong energy conditions.

Bali (1984, 1985) investigated solutions with viscous magneto hydrodynamic matter sources using the ansatz $\zeta\theta = \text{constant}$. Barrow (1988) has considered a radiation dominated model with constant coefficient of bulk viscosity and positive spatial curvature. Gron (1990) investigated viscous inflationary models and pointed out that bulk and shear viscosities cause exponential decay of anisotropy while non-linear viscosity causes power-law decay of anisotropy. Zimdahl (1996) investigated that sufficiently large viscous pressure leads to inflationary behaviour. The effect of viscosity upon the expansion of the universe in an inflationary era has been investigated by many authors viz. Waga et al. (1986), Padmanabhan and Chitre (1987), Barrow (1987), Lima et al. (1988), Chimento et al. (1997), Maartens and Mendez (1997), Brevik et al. (2011).

The effect of bulk viscosity on cosmological models is also investigated by Saha (2005), Singh et al. (2000), Bali and Singh (2008), Bali et al. (2012), Ram et al. (2012), Brevik and Gron (2013).

Inflation is the extremely rapid expansion of the early universe by a factor of 10^{78} in volume driven by negative pressure vacuum energy density. Guth (1981) introduced the concept of inflation while investigating the problem of why we do not see magnetic monopoles today. He found that a positive energy false vacuum generates an exponential expansion of space according to general relativity. As pointed out by Guth (1981), inflationary models provide a potential solution to the problem of structure formation in Big-Bang cosmology like horizon problem, isotropy problem, flatness problem and magnetic monopole problems. Inflationary scenario for homogeneous and isotropic space-time (FRW model) has been studied by many authors viz. Linde (1982), Burd and Barrow (1998), La and Steinhardt (1989). Rothman and Ellis (1986) have pointed out that we can have solution of isotropic problem if we work with anisotropic metric that isotropizes in special case. Keeping in view of these investigations, Bali and Jain (2002), Bali (2011) investigated some inflationary cosmological models for flat potential in Bianchi Type I space-time. Recently Bali and Singh (2014) investigated LRS (Locally Rotationally Symmetric) Bianchi Type I inflationary model for stiff fluid distribution with variable bulk viscosity.

In the present investigation, we have investigated spatially homogeneous anisotropic Bianchi Type I inflationary model for barotropic fluid distribution with variable bulk viscosity. To get the realistic scenario, we have assumed the barotropic

condition $p = (\gamma-1)\rho$, $1 \leq \gamma \leq 2$, $\rho = 3H^2$, $\zeta \propto \rho^{1/2}$ as considered by Barrow (1988)

and Gron (1990) and conservation equation $T_{i;j}^j = 0$ is taken into account.

We find that spatial volume increases with time representing inflationary scenario.

The deceleration parameter $(q) < 0$ which shows that the model represents accelerating universe which matches with the result as obtained by Riess et al.(1998) and Perlmutter et al. (1999).

2. Metric and Field Equations

We consider spatially homogeneous Bianchi Type I metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \quad (1)$$

where A, B, C are metric potentials and functions of t alone. Einstein field equation is taken in the form

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \quad (2)$$

(in geometrized unit $8\pi G = 1 = c$)

with

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij} - \zeta \theta (g_{ij} + v_i v_j) + \partial_i \phi \partial_j \phi - \left[\frac{1}{2} \partial_\rho \phi \partial^\rho \phi + V(\phi) \right] g_{ij} \quad (3)$$

and

$$\frac{1}{\sqrt{-g}} \partial_i [\sqrt{-g} \partial^i \phi] = -\frac{dV}{d\phi} \quad (4)$$

Where ρ is the matter density, p the isotropic pressure, ζ the coefficient of bulk viscosity, θ the expansion in the model, g_{ij} the metric tensor, v^i the flow vector satisfying $g_{ij} v^i v^j = -1$, ϕ the Higgs field and V the potential.

The **Higgs field** is an invisible energy field, that exists everywhere in the universe.

The field is accompanied by fundamental particle called Higgs-Boson, which it uses to continuously interact with other particles. As the particles pass through the field, they are endowed with the property of mass.

Einstein field equation (2) together with (3) and (4) for the metric (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -p + \zeta\theta - \frac{\dot{\phi}^2}{2} + V(\phi) \quad (5)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -p + \zeta\theta - \frac{\dot{\phi}^2}{2} + V(\phi) \quad (6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -p + \zeta\theta - \frac{\dot{\phi}^2}{2} + V(\phi) \quad (7)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} = \rho + \frac{\dot{\phi}^2}{2} + V(\phi) \quad (8)$$

Equation (4) leads to

$$\ddot{\phi} + \dot{\phi} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = \frac{dV}{d\phi} \quad (9)$$

3. Solution of Field Equations

Equations (5), (6) and (7) lead to

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_4}{C} \left(\frac{A_4}{A} - \frac{B_4}{B} \right) = 0 \quad (10)$$

and

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A} \left(\frac{B_4}{B} - \frac{C_4}{C} \right) = 0 \quad (11)$$

Equation (10) leads to

$$\left(\frac{A_4}{A} - \frac{B_4}{B}\right)_4 + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0 \quad (12)$$

Thus we have

$$\frac{\left(\frac{A_4}{A} - \frac{B_4}{B}\right)_4}{\left(\frac{A_4}{A} - \frac{B_4}{B}\right)} = -3H \quad (13)$$

Similarly equation (11) leads to

$$\frac{\left(\frac{B_4}{B} - \frac{C_4}{C}\right)_4}{\left(\frac{B_4}{B} - \frac{C_4}{C}\right)} = -3H \quad (14)$$

where

$$\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = 3H \quad (15)$$

H being Hubble parameter.

Now conservation equation

$$T_{\mu;\nu}^{\nu} = 0$$

leads to

$$\frac{\partial}{\partial t}(T_4^4) + \frac{\partial}{\partial t}(\log \sqrt{-g})T_4^4 - \frac{1}{2} \frac{\partial}{\partial t}(g_{\alpha\alpha})T^{\alpha\alpha} = 0 \quad (16)$$

Thus we have

$$\dot{\rho} + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \dot{\phi} \ddot{\phi} + \dot{\phi}^2 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$$

$$-\zeta\theta\left(\frac{A^4}{A} + \frac{B^4}{B} + \frac{C^4}{C}\right) = 0 \quad (17)$$

Stein-Schabes (1987) has pointed out that inflation will take place if potential $V(\phi)$ has flat region. To find inflationary scenario, we consider flat region so that $V(\phi)$ is constant. Therefore equation (9) leads to

$$\dot{\phi}\ddot{\phi} + \dot{\phi}^2\left(\frac{A^4}{A} + \frac{B^4}{B} + \frac{C^4}{C}\right) = 0 \quad (18)$$

To get the deterministic solution, we assume that universe is filled with barotropic fluid distribution $p = (\gamma - 1)\rho$ and $\rho = 3H^2$, $\zeta = a\rho^{1/2}$ as considered by Barrow (1988) and Gron (1990). Using the above conditions, now equations (17) with (18) and

$$\zeta = \frac{1}{3}\rho^{1/2} \quad (\text{assuming } a = \frac{1}{3}) \quad (19)$$

lead to

$$6H\dot{H} + 9H^3\left(\gamma - \frac{1}{\sqrt{3}}\right) = 0 \quad (20)$$

which leads to

$$H = \frac{2}{\alpha t + \beta} \quad (21)$$

where

$$\alpha = \sqrt{3}(\sqrt{3}\gamma - 1) \quad (22)$$

Equation (14), (15) and (22) lead to

$$\frac{A^4}{A} - \frac{B^4}{B} = \beta_1(\alpha t + \beta)^{-6/\alpha} \quad (23)$$

$$\frac{B_4}{B} - \frac{C_4}{C} = \beta_2 (\alpha t + \beta)^{-6/\alpha} \quad (24)$$

where β_1, β_2 are constants.

From equations(23), (24) and (15), we have

$$A = \beta_3 (\alpha t + \beta)^{2/\alpha} \exp \left[\frac{2\beta_1 + \beta_2}{3(\alpha - 6)} (\alpha t + \beta)^{\frac{\alpha-6}{\alpha}} \right] \quad (25)$$

$$B = \beta_4 (\alpha t + \beta)^{2/\alpha} \exp \left[\frac{\beta_2 - \beta_1}{3(\alpha - 6)} (\alpha t + \beta)^{\frac{\alpha-6}{\alpha}} \right] \quad (26)$$

$$C = \beta_5 (\alpha t + \beta)^{2/\alpha} \exp \left[-\frac{2\beta_2 + \beta_1}{3(\alpha - 6)} (\alpha t + \beta)^{\frac{\alpha-6}{\alpha}} \right] \quad (27)$$

where $\beta_3, \beta_4, \beta_5$ are constants and $1 < \alpha < 6$.

The metric (1) leads to the form

$$\begin{aligned} ds^2 = & -dt^2 + \beta_3^2 (\alpha t + \beta)^{4/\alpha} \exp \left[2 \left\{ \frac{2\beta_1 + \beta_2}{3(\alpha - 6)} (\alpha t + \beta)^{\frac{\alpha-6}{\alpha}} \right\} \right] dx^2 \\ & + \beta_4^2 (\alpha t + \beta)^{4/\alpha} \exp \left[2 \left\{ \frac{\beta_2 - \beta_1}{3(\alpha - 6)} (\alpha t + \beta)^{\frac{\alpha-6}{\alpha}} \right\} \right] dy^2 \\ & + \beta_5^2 (\alpha t + \beta)^{4/\alpha} \exp \left[-2 \left\{ \frac{(2\beta_2 + \beta_1)}{3(\alpha - 6)} (\alpha t + \beta)^{\frac{\alpha-6}{\alpha}} \right\} \right] dz^2 \end{aligned} \quad (28)$$

After suitable transformation of coordinates, the metric (28) leads to

$$\begin{aligned}
ds^2 = & -\frac{dT^2}{\alpha^2} + T^{4/\alpha} \exp \left[2 \left\{ \frac{2\beta_1 + \beta_2}{3(\alpha-6)} T^{\frac{\alpha-6}{\alpha}} \right\} \right] dX^2 \\
& + T^{4/\alpha} \exp \left[2 \left\{ \frac{\beta_2 - \beta_1}{3(\alpha-6)} T^{\frac{\alpha-6}{\alpha}} \right\} \right] dY^2 \\
& + T^{4/\alpha} \exp \left[-2 \left\{ \frac{2\beta_2 + \beta_1}{3(\alpha-6)} \right\} T^{\frac{\alpha-6}{\alpha}} \right] dZ^2
\end{aligned} \tag{29}$$

4. Physical and Geometrical Features

The matter density (ρ), the isotropic pressure (p), the bulk viscosity (ζ), the expansion (θ), the shear (σ), the spatial volume (R^3), the deceleration parameter (q) for the model (29) are given by

$$H = \frac{2}{T} \tag{30}$$

$$\rho = \frac{12}{T^2} \tag{31}$$

$$p = (\gamma - 1)\rho = \frac{12(\gamma - 1)}{T^2} \tag{32}$$

where $1 \leq \gamma \leq 2$

$$\zeta = \frac{\rho^{1/2}}{3} = \frac{2}{\sqrt{3} T} \tag{33}$$

$$\theta = \frac{6}{T} \tag{34}$$

$$\sigma = \frac{(\beta_1^2 + \beta_2^2 + \beta_1\beta_2)}{\sqrt{3}(6-\alpha)T^{6/\alpha}} \quad (35)$$

$$R^3 = ABC = \ell T^{6/\alpha} \quad (36)$$

$$q = -\frac{\alpha(2\alpha-1)}{2} < 0 \text{ as } \alpha > 1. \quad (37)$$

The Higgs field (ϕ) is given by equation (18) as

$$\phi = \frac{L \alpha T^{\frac{\alpha-6}{\alpha}}}{(\alpha-6)} + M \quad (38)$$

where L and M are constants.

5. Energy Conditions

Following Kolassis et al. (1988), Chatterjee and Banerjee (2004), we discuss briefly weak, dominant and strong energy conditions in the context of inflationary scenario with the bulk viscosity for the model (29). We have

$$\begin{aligned} T_{00} &= \rho + \frac{\dot{\phi}^2}{2} + k, T_{11} = p - \sqrt{3}H^2 + \frac{\dot{\phi}^2}{2} - k \\ &= T_{22} = T_{33} \end{aligned} \quad (39)$$

where $V(\phi) = k$ (constant).

In the locally Minkowskian frame, the roots of matrix equations

$$\begin{aligned} |T_{ij} - r g_{ij}| &= \text{diag} \left[\left(\rho + \frac{\dot{\phi}^2}{2} + k - r, r + p - \sqrt{3}H^2 + \frac{\dot{\phi}^2}{2} - k, \right. \right. \\ &\left. \left. r + p - \sqrt{3}H^2 + \frac{\dot{\phi}^2}{2} - k, r + p - \sqrt{3}H^2 + \frac{\dot{\phi}^2}{2} - k \right) \right] \end{aligned} \quad (40)$$

give the Eigen values r of our energy-momentum tensor as

$$r_0 = \rho + \frac{\dot{\phi}^2}{2} + k, r_1 = -p + \sqrt{3}H^2 - \frac{\dot{\phi}^2}{2} + k = r_2 = r_3$$

The energy conditions for our model are

5.1 Weak Energy Conditions

$$r_0 \geq 0 \text{ leads to } \rho + \frac{\dot{\phi}^2}{2} + k \geq 0, r_0 - r_1 \geq 0 \text{ leads to}$$

$$\gamma\rho + \dot{\phi}^2 \geq \sqrt{3}H^2 \text{ as } p = (\gamma - 1)\rho \quad (41)$$

5.2 Dominant Energy Conditions

$$r_0 \geq 0 \text{ leads to } \rho + \frac{\dot{\phi}^2}{2} + k \geq 0, -r_0 \leq -r_1 \leq r_0 \text{ leads to}$$

$$\rho \geq p - \sqrt{3}H^2 + 2k \quad (42)$$

5.3 Strong Energy Conditions

$$r_0 - \Sigma r_i \geq 0 \text{ leads to } \rho + 3p + 2\dot{\phi}^2 \geq \sqrt{3}H^2 + 2k \quad (43)$$

If we group (42) and (43) then we have

$$\rho + 3p + 2\dot{\phi}^2 \geq 3\sqrt{3}H^2 + 2k$$

The reality condition $\rho + p \geq 0$ is violated for inflationary universe due to the presence of scalar field ϕ .

6. Conclusion

The spatial volume increases with time representing inflationary scenario. The matter density is initially large but decreases with time. The bulk viscosity prevents the matter density to vanish. Initially the model starts with a big-bang at $T = 0$ and the expansion decreases with time.

Later on, it represents accelerating universe which matches with the result as obtained by Riess et al. (1998) and Perlmutter et al. (1999). The decelerating expansion at the initial epoch provides obvious provision for the formation of large structure of universe. The formation of structures in the universe is better supported by decelerating expansion. Also the late time acceleration is in agreement with the observations of 16 type Ia supernovae made by Hubble Space Telescope (HST) (Riess

et al. 2004). Since $\frac{\sigma}{\theta} \neq 0$ in general, therefore anisotropy is maintained. However, the model isotropizes at late time. The Higgs field evolves slowly but the universe expands. The presence of bulk viscosity tends to increase the inflationary phase. The energy conditions as given by Kolassis et al. (1988), Chatterjee and Banerjee (2004) are discussed. The model (29) has Point Type singularity at $T = 0$ (MacCallum (1971)). We also observed that the stiff fluid case ($\gamma = 2$), dust distribution ($\gamma = 1$) and radiation dominated model ($\gamma = 4/3$) are having similar results.

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