On Special Relativity: Incompatibility of the Light Speed Postulate with the Coordinate Transformation Symmetry Assumption

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Abstract

The speed of light postulate is closely examined from the perspective of two inertial reference frames—unprimed ('stationary') and primed ('traveling')—in relative motion, revealing that the speed of light postulate actually requires length contraction with respect to the unprimed reference frame, and length expansion with respect to the primed frame. It is shown that when symmetry is imposed on the inverse length transformation(i.e., to make it exhibit the same length contraction from the perspective of the primed frame), the common length contraction factor becomes nothing but the Lorentz contraction factor γ. However, this would necessarily result in $\gamma = 1$, implying thatthe frames are being at rest with respect to each other, and thus refuting the special relativity predictions!When the coordinate's transformation symmetry assumption is applied on the direct transformation resulting from the light speed postulate—which is shown incompatible with this assumption—, the Lorentz transformation and its inverse are erroneously obtained; it is shown to be restricted to certain coordinate relations, resulted in mathematical contradictions, and thus demonstrated to be unviable.

Keywords: Special Relativity; Speed of Light Postulate; Coordinates Transformation Symmetry; Lorentz Transformation; Length Contraction; Length Expansion

1. Introduction

The Lorentz transformation, providing interrelation between the coordinates of two inertial reference frames in relative motion, forms the heart of the Special Relativity Theory.

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Einsteinmainly derived the transformation on the basis of two principles: 1- the principle of relativity, stating that the laws of physics are the same in all inertial reference frames, and 2- the speed of light principle, postulating that the speed of light in vacuum is invariant with respect to all inertial frames of reference(A. Einstein, 1905; A. Einstein, 1907).

Yet, another essential tool used in the Lorentz transformation derivation is that the direct and inverse transformations exhibitmutually symmetrical property; that is, the inverse transformation equation can be deduced from the direct one by swapping the coordinates and reversing the velocity sign. This is essentially the result of the isotropic property of space, combined with the first principle of the special relativity. This assumption is rather intuitive. However, in this paper, it is demonstrated that the speed of light principle deviates from this "law" of transformation symmetry. That is, the speed of light principle consequent direct transformationfrom the perspective of one frame is not symmetrical relative to the corresponding inverse transformationfrom the perspectiveof the other frame in relative translational motion with respect to the first frame. It is shown that this fact has a fatal outcome in regard to the coherence of the special relativity, in agreement with the findings of earlier studies(Kassir, 2014a, 2014b).

2. Fatal Consequence of the LightSpeed Postulateandthe Transformation Symmetry

Let $K(x, y, z, t)$ be a coordinate system attached to a reference frame K, and let $K'(x', y', z', t')$ be another coordinate system attached toa reference frame K' in relative translational motionat a uniform velocity v, with respect to K.

A light ray is emitted when the two frames are overlying at the instant of time $t = t' = 0$, from a point at the coinciding frame origins, in the relative motion direction.
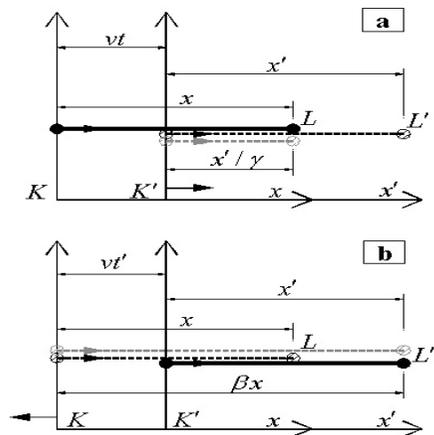

Fig.1 Light ray tip point path from the perspective of K (a) and K' (b)

According to the light speed principle, after period of time t with respect to $K,$ corresponding to t' with respect to K', has elapsed, the light ray tip will have travelled a distance $x = ct$ with respect to K, $x' = ct'$ with respect to K', where c is the speed of light in empty space.

Since, according to the special relativity's second postulate, the speed of light is the same with respect to both frames, the light ray trajectory drawn independently in K and K' would appear as shown in Fig. 1 in solid lines with equal lengths. However, the light ray tip point L' is actually perceived as point L (since L and L' represent the same event in each frame) with respect to K. Hence, the distance x' must be contracted with respect to K in order for point L' to coincide with point L. Suppose the distance x' is contracted by a factor of $(1/\gamma < 1)$, as shown in Fig. 1a with the gray dashed line, the following expression is inferred from Fig.1a, relative to K.

$$\gamma = \frac{x}{x - vt} = \frac{ct}{ct - vt} = \frac{1}{1 - \dfrac{v}{c}}. (1)$$

where vt is the distance travelled by K' with respect to K during the travel time t.

On the other hand, the light ray tip point L is actually perceived as point L', with respect to K' (since L and L' represent the same event in each frame). Hence, the distance x must then be expanded with respect to K' in order for point L to coincide with point L'. Suppose the distance x is expanded by the factor of $\beta > 1$, as shown in Fig. 1b with the gray dashed line. Hence, the following expression is inferred from Fig.1b, relative to K'.

$$\beta = \frac{x' + vt'}{x'};$$
$$\frac{1}{\beta} = \frac{x'}{x' + vt'} = \frac{ct'}{ct' + vt'} = \frac{1}{1 + \dfrac{v}{c}}. \qquad (2)$$

Equations (1) and (2)lead to

$$\frac{\gamma}{\beta} = \frac{1}{1 - \dfrac{v^2}{c^2}}. \qquad (3)$$

If we now impose that the length in K must—by the "law" of symmetry—be also contracted with respect to K' by the factor $(1/\gamma)$ (i.e., $\beta = 1/\gamma$)—contradicting the intrinsic length expansion relative to K'—then equation (3) reduces to

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}, \qquad (4)$$

which is the Lorentz contraction factor, in accordance with to the special relativity predictions.

However, comparing equations (1)and (4), the symmetry requirement results in

$$\frac{1}{1 - \dfrac{v}{c}} = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}};$$

$$\left(1 - \frac{v}{c}\right)^2 = 1 - \frac{v^2}{c^2}; \qquad (5)$$

$$1 - \frac{v}{c} = 1 + \frac{v}{c};$$

$$v = 0.$$

Or, the symmetry criteria $\beta = 1/\gamma$ leads,from Eqs. (1) and (2), to

$$1 - \frac{v}{c} = 1 + \frac{v}{c}, \text{ or } v = 0,$$

implyingthe reference frame must be at rest with respect to each other in order to satisfy the light speed principle and the transformation symmetry. It follows that the special relativity is deemed to be refuted.

3. Coordinate Transformation and Verification of Findings

Using Fig.1a, the following transformation is deduced from the perspective of an observer in K.

$$x = \frac{x'}{\gamma} + vt,$$
$$x' = \gamma(x - vt). \tag{6}$$

Similarly, Fig.1b leads to the following transformationfrom the perspective of an observer in K'.

$$x' = \beta x - vt',$$
$$x = \frac{1}{\beta}(x' + vt'). \tag{7}$$

Equations (6) and (7) lead to

$$\gamma x = x' + \gamma vt \tag{8}$$
$$\beta x = x' + vt'. \tag{9}$$

Dividing equation (8) by equation (9) we obtain

$$\frac{\gamma}{\beta} = \frac{x'}{x' + vt'} + \frac{\gamma vt}{\beta x};$$
$$\frac{\gamma}{\beta} = \frac{1}{1 + \dfrac{v}{c}} + \frac{\gamma v}{\beta c};$$
$$\frac{\gamma}{\beta}\left(1 - \frac{v}{c}\right) = \frac{1}{1 + \dfrac{v}{c}};$$
$$\frac{\gamma}{\beta} = \frac{1}{1 - \dfrac{v^2}{c^2}}, \tag{10}$$

verifyingequation (3).

It should be noted that the spatial transformation equations (6) and (7) deduced from the speed of light invariance are in conformance with the Galilean transformation for the limit $v \ll c$.

Now, if equations (6) and (7) were to be symmetrical, in accordance with the special relativity assumption of transformation symmetry, then

$$\gamma = \frac{1}{\beta};\qquad\qquad\qquad (11)$$

leadingfrom equation (10) to

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},\qquad\qquad (12)$$

contradicting the intrinsic length expansion relative to K' (β becomes <1),and equations (6) and (7) become the spatial Lorentz transformation equation and its inverse, respectively. However, $\beta < 1$ would result in shiftingpoint L to the opposite direction away from L' (Fig. 1b) making the same event occur in two different locations at the same time with respect to K', thus leading to an impossible occurrence.In fact, equations (1) and (12)would necessarilylead to $v = 0$.It follows that the special relativity prediction is deemed to be refuted.

4. The Special Relativity Blunder

Using the isotropic property of space, and the Special Relativity first postulate stating that the laws of physics are the same in all inertial reference frames, the coordinate transformation with respect to the unprimed frame K, given by equation (6)—obtained from the constancy of the speed of light postulate—would represent the inverse transformation (i.e., with respect to the primed frame K'), had we swapped in the equation the unprimed and the primed coordinates, and reverse the sign of the relative velocity (as K is traveling in the opposite direction with respect to K'). This will lead to the following transformation equation and its inverse.

$$x' = \gamma(x - vt); \qquad (13)$$
$$x = \gamma(x' + vt'). \ (14)$$

Obviously, equation (14) is inconsistent with the speed of light principle, as it is not in line with equation (7) required by this principle.

Now, dividing both sides of equations (13) and (14) by c, the speed of light, the following time transformation equations are obtained.

$$t' = \gamma t\left(1 - \frac{v}{c}\right), \ (15)$$

$$t = \gamma t'\left(1 + \frac{v}{c}\right). \ (16)$$

Substituting equation (15) into equation (16) leads after simple simplification to

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}. \ (17)$$

Replacing equation (16) in equation (13), and equation (15) in equation (14) ,return, respectively

$$x = \gamma\left[x'\left(\frac{1}{\gamma^2} + \frac{v^2 t'}{cx'}\right) + vt'\right];$$
and
$$x' = \gamma\left[x\left(\frac{1}{\gamma^2} + \frac{v^2 t}{cx}\right) - vt\right],$$

requiring $x' = ct'$ and $x = ct$, to yield the transformation equations (14) and (13), respectively. When this requirement (i.e., $x = ct$ and $x' = ct'$) is applied to equations (15) and (16), the following equations are returned

$$t' = \gamma\left(t - \frac{vx}{c^2}\right); \qquad (18)$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right). \qquad (19)$$

It follows that, equations (13), (14), (18), and (19), which are nothing but the Lorentz transformation equations, are restricted to $x = ct$ and $x' = ct'$, which leads to various contradictions.

In fact, when $t' = 0$, Lorentz transformation (18) leads to $t = vx/c^2$. But, as shown above, $x = ct$ in equation (18), yielding the contradiction $t = vct/c^2$, or $v = c$

Similarly, Lorentz transformation (19) can lead to a similar contradiction for $t = 0$ (i.e. $v = -c$).

Furthermore, substituting equation (18) into equation (19), returns

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right), \qquad (20)$$

which can be simplified to

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right). \qquad (21)$$

Since, as shown earlier, equations (18) and (19) require $x = ct$; $x' = ct'$, then equation (21) can be written as

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \qquad (22)$$

Now, for time $t' = 0$, the transformed t-coordinate with respect to K would be $t = vx/c^2$, according to equation (18). Consequently, for $t \neq 0$, equation (22) would reduce to

$$t\left(\gamma^2 - 1\right) = t\gamma^2,$$

yielding the contradiction,

$$\gamma^2 - 1 = \gamma^2, \ \text{ or } \ 0 = 1.$$

It follows that the conversion of the time coordinate $t' = 0$ to $t = vx/c^2$, for $x \neq 0$, by Lorentz transformation equation (18), is proved to be invalid, since it leads to a contradiction when used in equation (22), resulting from the Lorentz transformation equations for $t \neq 0$ (i.e. beyond the initial overlaid-frames instant satisfying $t = 0$ for $t' = 0$).

A similar contradiction is obtained by substituting equation (19) into equation (18), and applying equation (19) for the conversion $t = 0$; $t' = -vx'/c^2$.

In addition, substituting equation (13) into equation (14), yields

$$x = \gamma\left(\gamma\left(x - vt\right) + vt'\right);$$
$$x\left(\gamma^2 - 1\right) = \gamma v\left(\gamma t - t'\right);$$
$$x\left(\gamma^2 - 1\right) = \gamma vt\left(\gamma - \frac{t'}{t}\right). \qquad (23)$$

Since equations (13) and (14)—along with equations (18) and (19)—require $x = ct$; $x' = ct'$, equation (23) can be written as

$$x\left(\gamma^2 - 1\right) = \gamma vt\left(\gamma - \frac{x'}{x}\right). \qquad (24)$$

Now, for $x' = 0$, the transformed x-coordinate with respect to K would be $x = vt$, according to equation (13). Consequently, for $x \neq 0$, equation (24) would reduce to

$$x\left(\gamma^2 - 1\right) = x\gamma^2,$$

$$\gamma^2 - 1 = \gamma^2, \text{ or } 0 = 1.$$

It follows that the conversion of the space coordinate $x' = 0$ of K' origin to $x = vt$, at time $t > 0$, with respect to K by Lorentz transformation equation, is invalid, since it leads to a contradiction when used in equation (24), resulting from Lorentz transformation equations, for $x \neq 0$ (i.e. beyond the initial overlaid-frames position satisfying $x = 0$ for $x' = 0$).

A similar contradiction would follow upon substituting equation (14) into equation (13), and applying equation (14) for the conversion $x = 0$; $x' = -vt'$.

5. Conclusions

Considering two internal reference frames—unprimed and primed—in relative translational motion, the direct coordinate conversion factor and its inverse were easily deduced from the constancy of the speed of light principle, using simple diagrams for a light ray travel path from the perspective of each of the two frames. The direct length conversion factor was found to be in agreement with the corresponding special relativity prediction. However, the deduced inverse conversion factor was not symmetrical with respect to the direct length conversion that required that the space in the primed frame be contracted with respect to that of the unprimed frame, while the inverse length conversion factor showed the inverse relation (i.e., the length in the unprimed frame was expanded with respect to the primed frame).

It followed that, to achieve symmetry (i.e., length be mutually contracted with respect to both frames and by the same factor), the constancy of the speed of light principle required that the two frames be at rest with respect to each other, thus invalidating the special relativity predictions.Moreover, further analysis of the Lorentz transformation, following from the coordinate transformation symmetry assumption, showed fatal mathematical contradictions leading to its refutation.

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